MATH 54 - HINTS TO HOMEWORK 6

PEYAM TABRIZIAN

Here are a couple of hints to Homework 6! Enjoy! :)

Section 5.1: Eigenvalues and eigenvectors

Remember: To find the eigenvalues, calculate $det(A - \lambda I)$ and find the zeros of the resulting polynomial. To find a basis for the eigenspaces, find $Nul(A - \lambda I)$ for each eigenvalue λ that you found! Also, you should never get $Nul(A - \lambda I) = \{\mathbf{0}\}$

5.1.3. Calculate $A\mathbf{v}$, where A is the given matrix and $\mathbf{v}$ is the given vector.

5.1.17. Remember that the determinant of an upper-triangular matrix is just the product on the entries of the diagonal!

5.1.21.

(a) **F** ($\mathbf{x}$ has to be nonzero)
(b) **T**
(c) **T**
(d) **T** (depending on what you mean by easy and hard :))
(e) **F**

Section 5.2: The characteristic equation

5.2.15, 5.2.17. Remember that the determinant of an upper/lower-triangular matrix is just the product on the entries of the diagonal!

5.2.21.

(a) **F**
(b) **F**
(c) **T**
(d) **F** (-5 is an eigenvalue)

Section 5.3: Diagonalization

5.3.1, 5.3.3. If $A = PDP^{-1}$, then $A^k = PD^kP^{-1}$

Date: Friday, October 7th, 2011.

5.3.21.

(a) **T**
(b) **T**
(c) **F**
(d) **F**

Section 5.4: Eigenvectors and linear transformations

5.4.1. The matrix in question is $A = \begin{bmatrix} [\mathbf{T}(\mathbf{b_1})]_{\mathcal{D}} & [\mathbf{T}(\mathbf{b_2})]_{\mathcal{D}} & [\mathbf{T}(\mathbf{b_3})]_{\mathcal{D}} \end{bmatrix}$

5.4.3. Remember that $\mathbf{e_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = (1, 0, 0)$ etc.

5.4.9. For (b) Show $T(\mathbf{p} + \mathbf{q}) = T(\mathbf{p}) + T(\mathbf{q})$ and $T(c\mathbf{p}) = cT(\mathbf{p})$, for (c), calculate $T(1), T(t)$ and $T(t^2)$.

5.4.11. $A = \begin{bmatrix} [\mathbf{T}(\mathbf{b_1})]_{\mathcal{B}} & [\mathbf{T}(\mathbf{b_2})]_{\mathcal{B}} \end{bmatrix}$. That is, calculate $T(\mathbf{b_1})$ and $T(\mathbf{b_2})$ and express your answer in terms of $\mathbf{b_1}$ and $\mathbf{b_2}$.

5.4.17. This problem is **MUCH** longer than you think it is! To show A is not diagonalizable, calculate the eigenvalues of A, and find a basis for each eigenspace. If the sum of the dimensions of the bases is not equal to 2, then A is not diagonalizable.

Section 5.5: Complex eigenvalues

5.5.1, 5.5.3, 5.5.5. Just use the same technique you usually use to find eigenvalues and eigenvectors!

5.5.7, 5.5.9, 5.5.11. First, calculate $r = \sqrt{det(A)}$ (or take the length of the first row of A). Then factor out r from A and recognize the resulting matrix as a rotation matrix, i.e. find ϕ such that the remaining matrix equals to $\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}$

5.5.13, 5.5.15, 5.5.17. First, find the eigenvalues of A, and pick one of them. Then the first **ROW** of C consists the real and imaginary parts of the eigenvalue you picked. Then remember that the diagonal entries of C are the same, and the other entries are opposites of each other. Finally, to get P, find an eigenvector corresponding to the eigenvalue you picked, and then the columns of P are the real and imaginary parts of that eigenvector!